Instructions. Please answer all questions in the exam booklet. A portion of the marks awarded will be for the style and clarity of your answer. Marks will not be awarded in proportion to the length of an answer. Marks are indicated in boldface.

Do not use a hard pencil.

1. **(10) Definitions**
    a. Define the equality relation.
    b. In the context of the propositional logic, define a proof.

2. **(10)** Construct the truth table for $(S \vee (T \Leftrightarrow \neg R)) \Rightarrow ((S \vee T) \wedge \neg R)$.

3. **(20)** Show the following
    i. $(\forall x \forall y\ P(x,y)) \Leftrightarrow (\forall y \forall x\ P(x,y))$
    ii. $A \Rightarrow C \wedge D,\ B \Rightarrow D,\ B \Rightarrow C,\ (\neg A \wedge \neg B) \Rightarrow \neg P\ \ \vdash\ \ P \Rightarrow D$
    iii. Prove that $(A \Rightarrow B) \Rightarrow (\neg B \Rightarrow \neg A)$.
    iv. Show that it is not generally true that $(\forall x \exists y\ R(x,y)) \Leftrightarrow (\exists y \forall x\ R(x,y))$ by finding a relation R that always satisfies one side of the biconditional, but does not always satisfy the othr.

4. **Induction**
    a) **(5)** Give the definition of multiplication.
    b) **(10)** The associative property of addition is
        $(a + b) + n = a + (b + n)$.

        Using *only* the definition of addition, associativity, and mathematical induction, show that
        $0 + m = m + 0 = 0$.
    c) **(10)** Using familiar facts about arithmetic, and mathematical induction, show that the sum of the first n odd numbers is n^2. (A non-inductive proof is not acceptable.)
        $1 + 3 + 5 + \dots + (2n-1) = n^2$.

5. **(10)** Put the following formula into conjunctive normal form (that is, a conjunction of disjunctions of literals) using the algorithm we discussed in class.
    $\neg(Q \Leftrightarrow \neg P)$.

-End of Exam-